problem with Hatfield Model's approach is that it took a group relationship, one that holds only for the group, and applied it to each member of the group. This is called a fallacy of division. This logical error is common and has severe consequences. This error is a common source for stereotypical characterizations of ethnic, religious and gender groups that lead to various sorts of discrimination. In Attachment Section B, a graphical depiction of this error and its likely consequences are shown.

Related to this is the question is there any reason to believe that the group relationship might, nonetheless, hold individually. The answer is no. In the appendix II, Section C, we show that the only case where the common costs bear a direct linear relationship to joint costs is one where marginal costs are zero. This means that to believe the Hatfield Model's underlying model one must be willing to believe that the cost of supplying service to 10,000 extra customers is zero. Moreover, the Hatfield Model's formulation has the additional odd feature that, if volumes increase, then eventually the volume sensitive costs of joint production, which increase as volume increases, will exceed the volume insensitive costs of independent production. Consequently, production will take place independently using a technology having volume insensitive costs. Necessarily this means only one firm is needed to produce arbitrarily large amounts of any one service or element. Therefore were the Model's methods believed one need also believe that competition will fail and the current industry will be replaced by a group of natural monopolists.

The Model also suffers from problems in its choice of a sample, a single cross section of firms in a single year. The Model's limited sample is incapable of either supporting or refuting the analyses based on it. To determine whether or not its group relationship could be applied to a member of the group, it would need to use a panel of data, that is, multiple observations on each firm over time. It would need to do a pooled time-series cross-section analysis and test the hypothesis that the between-firm relationship is the same as the within-firm. A single cross-section cannot provide information on within-firm relationships because there is only one observation on each firm, whereas many more than the number of coefficients estimated are needed.[40]

Compounding the sample problem with the Model's analysis is its choice of methodology. Having shown that its group relationship cannot be applied to specific members of the group. We now show that the Model's method of obtaining the group relationship is also flawed. Regression analysis, like many other technical methods operates validly only in specific environments.

Statisticians and econometricians state the characteristics of the environments where regression analysis is valid in the form of assumptions. For example, a statistician might say a regression will give you the right answer provided none of the following occur. A conscientious practitioner of econometrics then checks the specific situation he or she is working in to make sure none of the required assumptions are violated. She might, for example, check to make sure that the independent variable, in Hatfield Model's case, direct costs, is uncorrelated with the error in the equation. If the independent variables are found to be correlated with the error, then regression analysis will lead to spurious results. Examples of

[40] An elementary but more complete explanation can be found in Greene, William H. (1993) *Econometric Analysis* 2nd Edition, Macmillan Publishing Company. New York. p. 444-480.

(continued...)

spurious results include the finding that as birth rates increased in Holland so too did the number of storks, leading to a conclusion that storks brought babies. In fact, when the citizens became wealthier, had more children and generated more garbage, more storks appeared in Holland to scavenge on the increased supply of garbage.. So the relationship is spurious, there is none between storks and babies, instead there is one between income and babies and income, through consumption and garbage, to the number of storks. The alleged relationship comes because both relations are positively related to income. However, a good trash collection policy or birth control policy would sever the relationship. The Model's group regression is of exactly this type.

Common costs neither cause nor are caused by direct costs: instead both are caused by the interaction of production with market forces. Specifically, a firm chooses inputs to minimize the total costs of production, thus the amount of direct and common costs are jointly determined. It can be found in any basic econometrics text that regression analysis is wrong when the dependent variable, here common costs, and the independent variable, here direct costs, are jointly determined.[41] The consequence is a simultaneous equation bias.

For an example of how misleading a regression with a simultaneous equations bias can be we need to can go back 50 years to the end of WWII (since then competent econometricians have known better than to make such errors). Then, the National Bureau of Economic Research

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(...continued)

[41] Greene, William H. (1993) *Econometric Analysis* 2nd Edition, Macmillan Publishing Company, New York p. 579.

The Hatfield Model uses ARMIS data to develop expense to investment factors. These factors are used to compute the expenses necessary for different network elements and their components. TELRIC of a network component multiplied by its corresponding expense factor yields the expense costs of that network element. Expense factors related to various components of many network elements are based on the amount of investment in that component—ARMIS expense divided by ARMIS investment. When certain expenses are deemed more sensitive to the number of customers, expense factors take the form of ARMIS expense divided by ARMIS reported number of lines.

As described in its documentation, the Release 3.0 still relies upon the same ARMIS calculated network-related expense factors as in Release 2.

> The two major categories under which network related expenses are reported by the ILECs are plant-specific operations expenses and non plant-specific operations expenses. The plant-specific expenses are primarily maintenance expenses. Certain expenses, particularly those for network maintenance, are strong functions of their associated capital investments. The Expense Module estimates these from historic expense ratios calculated from balance sheet and expense account information reported in each carrier's ARMIS report. These expense ratios are applied to the investments developed by the Distribution, Feeder, and Switching and Interoffice Modules to derive associated operating expense amounts. The ARMIS information used to perform these functions is contained in the "ARMIS inputs" worksheet, and the expense factors are computed in the "'95 Actuals" worksheet of the Expense Module.
> Other expenses, such as network operations, vary more directly with the number of lines provisioned by the ILEC rather than its capital investment.

issued a forecast and a prediction that as a consequence of the end of WWII and the return of the servicemen, the economy would be thrust back into a severe depression. That never happened. Milton Friedman, a Nobel Prize winner in Economics, and arguably also one of the great statisticians of that period, showed that the NBER had committed the very error we alluded to above, and as a consequence the prediction was fallacious[42].

## 4. Annual Expenses

The Hatfield Model develops expense estimates based upon ratios of *booked* expenses to investment. As discussed above, this approach is extremely problematic. Operating expense ratios based on historical investment are poor approximations of the forward-looking relationship. Consider, for example, an expense whose costs are unrelated to the underlying technology. As capital equipment becomes more (or less) productive, the expense to capital ratio changes, even though the absolute level of unit expenses does not.

The wire center switching example discussed earlier illustrates the pitfalls of using annual factors. By employing the unrealistic assumption that a LEC can buy switching at the initial prices, the model assumes that annual cost (which we understand include the generic upgrades) would be lower as well. In fact, the very report that Hatfield relies on to develop the switch model suggests that such additional costs may increase when switch vendors discount initial prices.

[42] Friedman, Milton (1957) *A Theory of the Consumption Function*, Princeton University Press. Princeton NJ

> Thus, expenses for these elements are calculated in proportion to the number of access lines supported.[43]

This way of calculating expense is an example of what's called, in statistical or econometric parlance, causal forecasting. When the Hatfield Model calculates an expense factor for a wire center building by dividing a 1995 ARMIS reported expense associated with buildings by 1995 ARMIS reported investments in wire center buildings, it is estimating a single parameter of a single equation regression model with one explanatory variable. In the case of the building expense, building investment is the sole explanatory variable. The equation is of the form E=*a*I, where E=expense, I=investment, and *a*=expense factor. Since the equation does not include an intercept term and, in the case of the building expense example, other explanatory variables such as the percentage of buildings leased, the single variable regression approach is simply inadequate.

The factor approach also suffers from the general problem that any decrease in an investment will cause a proportionate decrease in expenses. For example, if one LEC, for whatever reason, obtained a higher discount on its equipment, the model implies that it would enjoy lower out-of-pocket expenses, an implication that defies common sense.

A somewhat subtle, yet very serious error exists in the way the model computes expense factors. The Hatfield Model forecasts expenses, E, at time t+1 with the parameter *a* (expense factor) calculated at time t. It ignores that the equation used to predict E at time t+1 is based on

[43] "Model Description, Hatfield Release 3.0", Hatfield Associates, Inc., Boulder Colorado, page 56

a different independent variable than that used to estimate the parameter in the first place. The investment used to estimate the parameter *a* is the ARMIS reported investment and presumably includes embedded costs. At time t+1, the Model uses the independent variable of TELRIC calculated in the framework of the model. Again, the ARMIS reported investment costs and the forward-looking costs computed by the Model are not inter-changeable variables. As the model estimated TELRIC understates the value of the independent variable this misspecification biases the forecast of expenses downward.

## E. Proper Factor Estimation Method

To conclude Section II we address the subject of econometric or statistical estimation of expense factors. Before doing so we note that the problem with the factor approach from an econometric point of view is that each of the factors must be constrained so that total costs satisfy the required cost conditions above. While homogeneity is easy enough to enforce, the requirement that costs be increasing in all outputs and that variable cost be decreasing in all fixed factors can be difficult to impose without limiting the types of substitution the cost function can exhibit. For example, forcing the individual factors to increase as outputs will do this however there is nothing theoretical that requires any particular factor to either increase or decrease in output except that in the aggregate the cost function itself must. Rather than deal with that problem here, we abstract from those problems to suggest some simple, practical methods of estimating the factors.

For simplicity sake let us assume there is only one factor of interest, one of either the expense to dollar investment type or of the expense to physical quantity type. Extension to a complete set will be analogous to extending single equation regression or time series methods to multiple regression relations e.g., Seemingly Unrelated Regressions, Three-Stage-Least Squares or Vector-Auto Regressions.

If input prices change at roughly the same rate along with outputs, the factors might change smoothly as well. If so, a time series type of adjustment model might be able to predict future or forward looking factors from data on past factors, input price changes, output changes and fixed factor changes. Under the conditions just mentioned one might totally differentiate $F_1(y, w, K, t)$ with respect to time to obtain $F_1(\dot{y}, \dot{w}, \dot{K}, t) = a_1(t)\dot{y} + a_2(t)\dot{w} + a_3(t)\dot{K} + a_4(t)$, this might lead us to expect a time series relationship for $F_1(y, w, K, t)$ of the form

$$F_1(y, w, K, t) = a_1(t)y + a_2(t)w + a_3(t)K + a_4(t)$$

or in regression form

$$F_t = a_1(t)y_t + a_2(t)w_t + a_3(t)K_t + a_4(t) + \varepsilon_t .$$

Where the $a_j(t) = \sum_{i=1}^{I_j} a_j^i(t)L^i$ is a time varying polynomial lag operator. Such a specification is general enough to represent any form of interest. For example, time varying parameters regression, state-space regression, ARIMA, or simple regression.

A forward-looking expense factor might be estimated by such a time varying parameters regression model. Indeed, there is no reason why it might not equally well be represented by an ARIMA or by an auto-regressive-distributed lag or by a distributed lag. Thus one might be able to reduce the problem of estimating forward-looking expense factors to a simple time series problem. Indeed, one could imagine properly cleaned up ARMIS data might be used as a basis for estimating these factors. The reason such an approach would be forward looking even though it is based on historical data is that the factor used is not a mere average of past factors but a prediction of a future factor that adjusts for anticipated and historical changes in input prices, through its dependence on input prices, on technology, through its dependence on time, and so on. If one were lucky, and all variables followed a simple time series, the factors might have simple ARIMA representations, eliminating the need to collect direct data on input prices and the like.

The drawback to such an outcome would be that details of substitution could not be investigated. However, for estimation of forward looking costs using investment predictions derived from engineering process models that would be adequate. To complete the program all factors would, of course need to be estimated jointly.

Taking this last idea a step further, one might consider a panel of factors, that is a pooled cross section time series of factors for different firms in different time periods. With luck, change might be regular enough that all the data needed might be publicly available. One could then predict or forecast the forward looking factors according to the regression results. Obviously this idea needs to be fleshed out and some research done to check it. However, it does show promise given the success of time series methods for predicting Total Factor Productivities.

# Section III Structural Deficiencies

In this section, we address the most fundamental of the many theoretical or structural problems accompanying the Hatfield Model. By design, the Hatfield Model is *not* a valid cost model because it fails internal and external consistency checks required of any cost model. Whether estimating costs using a pure econometric approach, a pure engineering approach or some hybrid approach, common practice model building requires internal and external validation of a model. Our internal checks demonstrate that the Hatfield Model is theoretically incapable of representing the minimum cost of producing telecommunications services using the most efficient forward looking technology. Our external checks produce similar conclusions and confirm that the model produces results with no credibility.

## A. Valid Cost Models

The Hatfield model is not a valid economic cost model because it fails the internal validity check required of any cost model. This is more than just a theoretical point. Failure to satisfy these checks means that the Hatfield Model cannot represent the minimum cost of producing outputs using the most efficient forward looking technology. In Attachment Section A, we show this and also show that any numbers the Hatfield model produces purporting to be TS/TELRICs are biased in an unknown direction, meaning that they are not even correct on average. This makes them useless for even the minimal task of providing upper and or lower bounds for prices. Further, we will show that the underlying approach is so flawed as to render the Model impossible to fix without a complete overhaul, starting with the basic conceptual approach and ending with data requirements.

The primary purpose of a cost model is to answer the question "What is the minimum cost of producing a stream of outputs using the most efficient forward looking technology and facing a perhaps uncertain stream of input prices?" To use a cost model to calculate a TS/TELRIC for a product, one calculates the minimum cost of doing business as usual and subtracts from that the minimum cost of doing business if a product line were dropped from production. Both components of this difference should be dynamic cost functions, not costs calculated only for the year in question, but costs calculated over the optimal planning horizon of the firm. Single period static cost functions are totally inappropriate.

## B. Internal Validity Checks

A valid cost model shows the relationship between the minimum cost of producing a flow of services using the most efficient technology, given a set of expected input prices, starting today and flowing into the future as far as the firm's optimal planning horizon. Specifically, for input prices and output levels in each year of the planning period, it shows the minimum present discounted value of producing those levels of outputs.

As a consequence of this minimization, costs functions and cost models *necessarily* satisfy a set of mathematical properties which can be found in a first year graduate textbook such as 'Microeconomic Analysis' by Hal Varian.[44] Rather than a complete listing of them, we will discuss two that the Hatfield Model clearly violates. The first is linear homogeneity in prices; this means if all prices are increased proportionately, then total costs will increase by the

[44] Varian, Hal R. *Microeconomic Analysis, Third Edition*, New York: Norton, 1992.

same proportion. The second is the derivative property. An easily understood form of the derivative property is this: the percentage increase in total costs as a consequence of a one hundred percent increase in the price of an input, i.e. labor, loops, wire, and the like, will be exactly equal to the share of total costs directly attributable to that input. So if cable of a certain grade comprises 10% of total costs and its price rises 100%, then total costs should rise 10% as a consequence.

To test the linear homogeneity assumption, we increased all the input prices in the Hatfield model through the front-end user interface by 10%, using their default GTE California data as benchmark. A valid cost structure should yield an increase in TS/TELRICs of 10% as well.[45] The results of this first test are presented below in Table 6, and can be seen to yield increases of roughly 8%—a number 20% lower than it should be.

[45] This result is proven in Attachment 2.

## Table 6

## Comparison of Hatfield Version 2.2 Release 2 TSLRIC Results
## GTE California

| | GTE Base Case (1) | Costs with All Input Prices Increased 10% (2) | Percent Change (3) [(2)-(1)]/(1) | Percent of Total Cost of Network Elements (Base) (4) |
|---|---|---|---|---|
| Loop Distribution | $6.75 | $ 7.34 | 8.8% | 42.54% |
| Loop Concentrator/Multiplexer | $1.75 | $ 1.91 | 9.0% | 11.07% |
| Loop Feeder | $2.63 | $2.85 | 8.5% | 16.52% |
| Local Switching | $1.10 | $ 1.18 | 7.0% | 6.84% |
| Other | | | | 23.04% |
| Operator Systems | $6,876,152 | $ 7,475,702 | 8.7% | |
| common Transport | $0.0027286 | $ 0.00293 | 7.4% | |
| Dedicated Transport | $4.07 | $ 4.36 | 7.2% | |
| Signaling Link Transport | $52.28 | $56.04 | 7.2% | |
| Signaling Transfer Points | $0.0000769 | $ 0.00008 | 4.0% | |
| Service Control Points | $0.0010559 | $ 0.00113 | 7.0% | |
| Tandem Switching | $0.0018684 | $0.0020 | 7.0% | |
| **Total Cost of Network Elements** | **$730,471,465** | **$ 791,268,930** | **8.3%** | **100.0%** |

The test clearly indicates that the Hatfield Model does not fulfill the linear homogeneity test. We also verified that the Release 3.0 fails the same test (See Table 7 below for the results).

**Table 7**

**Comparison of Hatfield Model Release 3.0 TSLRIC Results GTE California**

| | GTE Base Case | Costs with All Input Prices Increased 10% | Percent Change | Percent of Total Cost of Network Elements (Base) |
|---|---|---|---|---|
| | (1) | (2) | (3) [(2)-(1)]/(1) | (4) |
| NID | $0.72 | $0.79 | 9.4% | 4.3% |
| Loop Distribution (all) | $5.94 | $ 6.51 | 9.5% | 35.8% |
| Loop Concentrator( all) | $2.77 | $ 3.01 | 8.7% | 16.7% |
| Loop Feeder (all) | $3.21 | $3.51 | 9.5% | 19.3% |
| Total Loop (all) | $12.64 | $ 13.82 | 9.3% | 76.2% |
| **Total Cost of Network Elements** | **$887,151,410** | **$956,904,158** | **7.73%** | **100.0%** |

A theoretical explanation of the necessary failure of both versions of the model to satisfy linear homogeneity and the derivative property is presented in Attachment Section A. In that presentation, we show that the Model's dependence on the use of constant expense factors to account for expenses is the root cause of its problem. By employing expense per pair-mile of cable as it does for installation and structure expenses it necessarily violates linear homogeneity.

For example, by assuming constant expense per dollar of investment factor as it does for most of the rest of expenses, it almost certainly violates the derivative property of cost functions. Regardless of the source or reason for the error, the fact that the Model produces wrong results is incontrovertible. And to emphasize the consequences of the error we once again point out that any cost function or cost model that fails even one of the criteria required of a cost function, whether as stated above or found in a text, cannot represent the minimum cost of producing services using the most efficient forward looking technology.

## Section IV Release 3.0

Although time constraints prevented us from examining Release 3.0 on the scale that had been done for Version 2.2 Release 2, we believe that the new release is likely to be no more a competent cost model than its predecessor. We base this opinion on two facts: 1) all of economic deficiencies, including modeling problems of the model, are intact in the new release and 2) the model is still not a structurally valid cost model and there is still no external verification to support its use.

### A Economic Deficiencies.

The model's basic premise of building new networks based on raw geographic and demographic data remains unchanged in the new release. Adding to problems arising from incorrect data and their usage, the Release 3.0 suffers from the incorrect "scorched-node" approach of building networks. It still ignores the economic dynamism and uncertainty of the real world. Furthermore, it disregards physical constraints to building a perfectly streamlined loop network and continues to subscribe to the "pick and choose" method of establishing costs for any particular ILEC—thereby erroneously giving each ILEC the benefit of obtaining the best input prices observed across the industry and across time.

Even its attempt to correct a particular economic modeling problem (pointed out in Section II D) of basing annual investment cost on return on net plant at the end of the year is done incorrectly. The Release 3.0 bases annual investments on net plant at mid-year which is analogous to a lending bank receiving mortgage payments on a house at the beginning of the year based on the predicted mortgage balance on July 1 of that year. The home owner would pay no interest on the unpaid portion of mortgage for the first six months of every year of the term on his mortgage. Although the homeowner would pay extra interest on the latter half each year, his total cost would be reduced by basing mortgage payments on the mid-year balance of his home loan.

## B. Validity of the Model.

As we discussed in relation to its predecessor, Release 3.0 does not fulfill the external or internal validity requirements of a cost model. Its basic model structure has not been modified, leading to its necessary failure on internal consistency checks. To our knowledge the authors of the model have still not attempted to externally validate the model. The results of the Release 3.0 for GTE companies in California, Washington and Texas reveal that the model it its latest form does not produce results significantly different from Version 2.2 Release 2 (see Executive Summary). We have already established that the Version 2.2 Release 2 does not satisfy external validity requirements (see Section II B).

## Conclusion

There are numerous reasons not to use the Hatfield Models (Version 2.2 Release 2 and Version 3.0) to determine TS/TELRICs and none to support their use. One of the most vexing problems is that neither of the Hatfield Models have ever been tested against real data as might be expected of any model of any type. Trying to use a model in spite of this fact is a little like asking paying customers to fly on a plane which has never before flown or even tested.

As an added insight to the problem of using a model that has not been verified with actual data, consider the following example. Suppose that the IRS decides to simplify its analysis of all of the paper work associated with reporting and verifying tax payers' income. To make the process easier, the IRS decides to create a model that estimates how much income from employment and investment a person earns each year. The model is simply based on assumptions about how much a person should be earning based on the tax payer's age and the number of years of schooling that the person has completed. To use this model, the IRS enters the person's age and number of years of schooling and lets the model derive an estimate of income which is used in place of any reported income. Despite valid criticisms of consumer groups and without taking the time to validate what the model predicts with actual income data, the IRS then uses this model to estimate a tax payer's income and taxes the person accordingly. We would hope everyone recognizes this as a ludicrous idea, but this is an exact analogy of what the Hatfield Models are doing to ILECs.

Beyond lack of external verification and empirical validity, explicit economic and conceptual flaws were identified that make the Models unlikely to produce any useable numbers. The Models are static rather than dynamic which gives rise to, among other things, fill factors that are too high. An equally troubling aspect of the Models is their fundamental assumptions that the telecommunication industry will not face increased market uncertainty and that LECs have had, and will continue to have, perfect foresight of all market conditions.

The Models do not even satisfy the minimum criteria required of properly constructed cost models—that increasing all prices by a common proportion must increase TS/TELRICs by exactly the same amount. In addition, there are other fundamental flaws in the Hatfield Models that we have identified: (1) they model the cost of no realistic local service provider and certainly not the incumbent LECs who will actually sell the unbundled elements and (2) particular inputs and processes appear to systematically understate the costs of network elements.

The Hatfield Model developers defend their costs by arguing that any difference between the costs of their model and costs reported by the LECs (either accounting costs that are required by law and by regulators or the cost produced by LEC incremental cost models) represent the costs of over-investment. For example, Version 2.2 Release 2 of the Model claims that about half of the LEC's current plant represents over-investment. Our preliminary analysis of Release 3.0 indicates it makes the same claim as its predecessor.

Apart from the fact that this label is entirely meaningless, since the Models call over-investment anything with which they do not agree, and that the Models' estimate of the so-called gap is fatally flawed by the theoretical and measurement problems, it defies common sense to believe that over-investment of this degree could take place.[46] Regulators (both at the federal and state level) would have to have been quite derelict in their public responsibilities for such this event to have occurred, an unlikely event given the scrutiny the telecommunications industry receives.

[46] Some of the gap between book investment and forward looking investment could represent the effect of the decline in prices for facilities such as end office switches. The fact that current prices recover some of these costs is entirely consistent with the economic fact that with technological change, no firm could survive by charging prices that completely reflect the decline in new equipment prices.

## Attachment

In this Attachment we demonstrate that the Hatfield Model violates the derivative property and that it produces biased TS/TELRICs. We begin with a brief discussion of the factor approach to cost model building and Hatfield Model's misuse and misunderstanding of it.

### Section A

A majority of the technical errors in the Hatfield model arise from its authors' dependence on the use of charge factors to handle everything from expenses to common costs. In this section we will give a justification for a form of the charge factor approach and then use this as a basis from which to analyze the Hatfield model. A mathematical function $C(y, w, K)$ is said to be a cost function if it represents the minimum cost of producing a set of outputs y, such as subscriber lines, minutes of use and the like (we will treat fill factors later ), when the prices for a set of inputs x are w, e.g. wages, materials, et cetera, using physical units of machinery, switches, conduit and so on usually referred to as capital, K. Formally,

$$C(y, w, K) = \min_{x} \{w^{T}x | F(y, x, K) \leq 1\}$$

where F(y,x,K) is a traditional distance function and the locus of points $\{(y, w, K) | F(y, x, K) = 1\}$ represent the most efficient combinations of inputs, that is, the best practice engineering method of combining inputs and outputs.

As a consequence of being the minimum cost of producing y using K, when input prices are w, a cost function satisfies the following mathematical properties, which can be found in any graduate microeconomic theory text:

1. It is non-decreasing in y
2. It is non-decreasing in w
3. It is non-increasing in K
4. It is convex in w
5. It is linear homogeneous in w
6. The inputs demand functions are given by $x(y, w, K) = \dfrac{\partial C(y, w, K)}{\partial w}$

7. There is one and only one distance function generating $C(y,w,K)$; equivalently there is one and only one technology consistent with C.
8. In addition, conditions 5 and 6 together imply that the input demand functions $x(y,w,K)$ are zero degree homogeneous in input prices w.

## 1. The Charge Factor Approach

Below we delineate how to generate a cost function in the charge factor form that seems popular with accountants and which is the prevalent form in which public utilities model costs. Let $x=\{x_1, x_2 \Lambda, x_m\}$ where the $x_i$ represent mutually exclusive subsets of inputs, then a cost function can always be written as

$$\begin{aligned} C(y,w,K) &= w_1^T x_1(y,w,K) + w_2^T x_2(y,w,K) + \Lambda + w_m^T x_m(y,w,K) \\ &= E_1(y,w,K) + w_2^T x_2(y,w,K) + E_3(y,w,K) + E_4(y,w,K) + \Lambda + E_m(y,w,K) \end{aligned}$$

Where $E_i(y,w,K)$ is the optimal (cost minimizing) expenditure on the ith group of inputs. Note that as a consequence of conditions 5, 6 and 8 above, we have:

9. The group expenditure or cost functions are first degree homogeneous in w.

Assume that $x_2$ is a scalar quantity such as installed lines, then the above equations can be rewritten as

$$\begin{aligned} C(y,w,K) &= \left(\frac{E_1(y,w,K)}{x_2(y,w,K)}\right) x_2(y,w,K) + w_2 x_2(y,w,K) + \left(\frac{E_3(y,w,K)}{E_4(y,w,K)}\right) E_4(y,w,K) + E_4(y,w,K) + \Lambda \\ &\quad + E_m(y,w,K) \\ &= \left(\frac{E_1(y,w,K)}{x_2(y,w,K)} + w_2\right) x_2(y,w,K) + \left(\frac{E_3(y,w,K)}{E_4(y,w,K)} + 1\right) E_4(y,w,K) + \Lambda + E_m(y,w,K) \\ &= F_1(y,w,K) x_2(y,w,K) + F_4(y,w,K) E_4(y,w,K) + \Lambda + E_m(y,w,K) \end{aligned}$$

Where we have introduced charge factors $F_1$ and $F_4$ defined as

$$F_1(y,w,K) = \left(\frac{E_1(y,w,K)}{x_2(y,w,K)} + w_2\right)$$

$$F_4(y,w,K) = \left(\frac{E_3(y,w,K)}{E_4(y,w,K)} + 1\right)$$

As a consequence of 8 and 9 above we have that $F_1(y, w, K)$ is first degree homogeneous in w and $F_4(y, w, K)$ is zero homogeneous in w. Any cost function therefore can be represented in the charge factor form provided the factors and the terms to which they are applied satisfy the conditions above.

## 2. Biases in Hatfield' Model's Factor Approach

Let $x_{ci}$ be the physical quantity of input i in input group c and let $p_{ci}$ be its price. Let $E_{ci}$ be expenditure on $x_{ci}$ and let $E_{si}$ be the expenditure on other inputs associated with $x_{ci}$, that is expenses.

### a. The Hatfield Model Violates the Derivative Property

The loop cost part of the Hatfield model may be represented as

$$C = \sum_{i=1}^{n} (p_{ci} x_{ci}) \left[ 1 + \left( \frac{E_{si}^{o}}{E_{ci}^{o}} \right) \right].$$

The derivative property of cost functions requires that the derivative of a cost function with respect to an input price give the optimal amount of the input.[47] Thus, the derivative of C with respect to $p_{ci}$ should give $x_{ci}$. Symbolically this is.

$$\frac{\partial C}{\partial p_{ci}} = x_{ci}.$$

Unfortunately, direct calculation of the partial derivative of the Hatfield model yields

$$\frac{\partial C}{\partial p_{ci}} = x_{ci} \left[ 1 + \left( \frac{E_{si}^{o}}{E_{ci}^{o}} \right) \right]$$

which is an over statement of $x_{ci}$ by a factor of

[47]We use the level form of the derivative property here rather than the proportional or logarithmic derivative form we used in the text, because the level form has easier mathematics.

$$\left[\left(\frac{E_{si}^{o}}{E_{ci}^{o}}\right)\right].$$

**b. Hatfield violates linear homogeneity**

This follows from checking condition 9 above. By that condition, factors that are ratios of expenses to physical quantities must be homogeneous of degree one. However, Hatfield assumes the expense to cable factor is constant. Constants do not change and so do not double when all input prices change, thus they fail homogeneity.

**c. Hatfield TS/TELRICs Are Biased**

For simplicity, assume only expenditures on cable and expenses. The results are exactly the same with switching and expenses except the notation is more elaborate and difficult to follow. The Hatfield Model gives a cost function of the following form:

$$\begin{aligned} C^* &= \sum_{i=1}^{n}(p_{ci}L_{ci})\left[1+\left(\frac{E_{si}^{o}}{E_{ci}^{o}}\right)\right] \\ &= \sum_{i=1}^{n}(E_{ci})\left[1+\left(\frac{E_{si}^{o}}{E_{ci}^{o}}\right)\right]. \end{aligned}$$

The cost minimizing cost function is

$$C = \sum_{i=1}^{n}(E_{ci} + E_{si}).$$

Use the difference calculus to obtain Hatfield TS/TELRIC and the true TS/TELRIC.

For the Hatfield Model,

$$\Delta C^* = \sum_{i=1}^{n}(\Delta E_{ci})\left[1+\left(\frac{E_{si}^{o}}{E_{ci}^{o}}\right)\right],$$

for the true model

$$\Delta C = \sum_{i=1}^{n}(\Delta E_{ci} + \Delta E_{si}).$$

Taking the difference between the terms gives

$$\Delta C - \Delta C^* = \sum_{i=1}^{n}\left(\Delta E_{ci} + \Delta E_{si} - (\Delta E_{ci})\left[1+\left(\frac{E^o_{si}}{E^o_{ci}}\right)\right]\right)$$
$$= \sum_{i=1}^{n}\left(\Delta E_{si} - (\Delta E_{ci})\left(\frac{E^o_{si}}{E^o_{ci}}\right)\right)$$
$$= \sum_{i=1}^{n} E^o_{si}\left(\frac{\Delta E_{si}}{E^o_{si}} - \frac{\Delta E_{ci}}{E^o_{ci}}\right).$$

Dividing by $\Delta y$, multiplying and dividing by y and rearranging terms gives

$$\frac{\Delta C - \Delta C^*}{\Delta y} = \sum_{i=1}^{n}\frac{E^o_{si}}{y}\left(\frac{\Delta E_{si}}{E^o_{si}}\frac{y}{\Delta y} - \frac{\Delta E_{ci}}{E^o_{ci}}\frac{y}{\Delta y}\right)$$

which is the bias in the incremental costs. The bias is then a weighted sum of the differences between installation and structure expenditure elasticities and the cable expenditure elasticities.

### d. Valid TS/TELRICs Must Be Linear Homogeneous in Input Prices

As discussed above, total cost functions must be first degree (or linear) homogeneous in input prices. This means if all input prices are increased by the same percent, say 10%, than total costs will increase by the same percent, in this case 10%. In this section we show that TS/TELRICs must satisfy the same requirements. We state the result as a Lemma:

TS/TELRICs are linear homogeneous in input prices.

Proof:

Let the total cost of providing n services at levels $y_1,...y_n$, with m inputs which have prices $w_1,...,w_m$ be denoted $C(y_1,...y_n, w_1,...,w_m)$. The TS/TELRIC for service 1 is given by

$$TS/TELRIC_1(y_1,...y_n,w_1,...,w_m) = C(y_1,...y_n,w_1,...,w_m) - C(0,y_2,...y_n,w_1,...,w_m).$$

Where $C(0, y_2, ..., y_n, w_1, ..., w_m)$ is the minimum cost of dropping the production of service one entirely while keeping the levels of all other outputs at their previous values. Thus, both $C(y_1,...y_n, w_1,...,w_m)$ and $C(0,y_2,...y_n, w_1,...,w_m)$ satisfy the linear homogeneity requirements,

$$\lambda C(y_1,\Lambda\ ,y_n,w_1,\Lambda\ ,w_m) = C(y_1,\Lambda\ ,y_n,\lambda w_1,\Lambda\ ,\lambda w_m)$$
$$\lambda C(0,y_2,\Lambda\ ,y_n,w_1,\Lambda\ ,w_m) = C(0,y_2,\Lambda\ ,y_n,\lambda w_1,\Lambda\ ,\lambda w_m)$$